Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 51-29 (2018) 38–43

# Adaptive Optimal Control of Wave Energy Converters

**Siyuan Zhan,** * **Bin Wang,** ** **Jing Na,** ** **Guang Li** *

* *School of Engineering and Materials Science, Queen Mary University of London, E1 4NS, London, UK (e-mail: g.li@qmul.ac.uk; s.zhan@qmul.ac.uk)*

** *Faculty of Mechanical & Electrical Engineering, Kunming University of Science & Technology, Kunming 650500,China (e-mail: najing25@163.com; wangbin2173@126.com).*

Abstract: This paper proposes a hierarchical adaptive optimal control framework for wave energy converters (WECs) to improve their energy conversion efficiency and reduce the required modeling effort to facilitate the control design. Since the WEC dynamics vary significantly at various operation scenarios with different sea states, an efficient adaptive parameter estimation (APE) algorithm is employed to online update several critical WEC model parameters (e.g. the radiation force and excitation force generation coefficients). Based on the updated model, the predesigned non-causal optimal controller can maintain its efficacy. Thus the proposed method combines the strength of optimal control for generating maximum energy output and APE in coping with model parameter variations. Simulation results demonstrate that the proposed optimal control with APE can cope with the model mismatch and variations effectively.



*Keywords:* Wave energy converter, adaptive parameter estimation, optimal control.

## 1. INTRODUCTION

Ocean waves provide vast, persistent and spatially concentrated energy compared with other renewable energy resources, e.g. solar and wind energies (Glendenning, 1977; Clément and et al, 2002). However, current wave technology is still immature for commercial use because the low energy extraction rate and the high risk of device damage can cause much higher unit cost of generated electricity than fossil fuels and even other relatively mature renewable energies (Bull and Ochs, 2013).

Apart from mechanical designs, controller design also plays an important role in improving the energy conversion rate and retaining safe operation of WECs. Some control strategies have been specifically developed for the WEC control problem, such as latching control (Babarit and Clément, 2006; Korde, 2002), phase control (Budal et al., 1981), and declutching control (Babarit et al., 2009). The optimal control of WECs is essentially different from the conventional optimal control problems (Zhan et al., 2016), (Falnes, 2002). Conventional optimal control strategies cannot be directly used for the WEC control since a WEC is subject to the persistent wave excitations (excitation force from incoming waves) and as a result there are no equilibria or fixed references for tracking. Recently, some optimal control strategies for the WEC control problem have been developed, e.g. a non-standard linear quadratic Gaussian (LQG) control (Scruggs et al., 2013), the noncausal control (Nielsen et al., 2013), and model predictive control (MPC) (Li et al., 2012).

However, the above optimal control strategies for the WEC control problem are mainly developed for a typical sea state. The dynamic model of a WEC can change significantly in different sea states due to the frequency dependent terms for calculating the radiation force and excitation force. A comprehensive model which can represent the dynamics for a range of sea states can cause computational issues for the controller design and the real-time implementation. Motivated by this fact, we propose to use a simplified WEC model whose dynamic parameters can be adaptively identified online. Based on this time-varying model, an optimal controller will be updated online to maintain satisfactory control performance over a range of sea states, while not causing significant computational load.

This paper will propose an online adaptive parameter estimation (APE) approach to estimate the damping terms associated with the radiation force and the wave excitation force and then incorporates it into an optimal controller. Although many algorithms have been proposed for APE, e.g. gradient and least squares, they are not suitable for WEC systems since the modeling uncertainties of WECs are varying. The APE for time-varying parameters is still a challenging topic in the filed (Ding et al., 2016), (Samandeep Dahliwal, 2014), (Na et al., 2015b). In our latest work (Na et al., 2015a), a novel APE framework has been proposed, where the adaptive law is driven by the parameter estimation error to achieve exponential or even finite-time error convergence. Hence, the contribution of this paper is to further tailor this new APE approach (Na et al., 2015a) to estimate time-varying parameters in the WEC systems, and then validate its efficacy when the estimated parameters are incorporated into a recently proposed linear noncausal optimal control (Zhan and Li,


10.1016/j.ifacol.2018.09.466

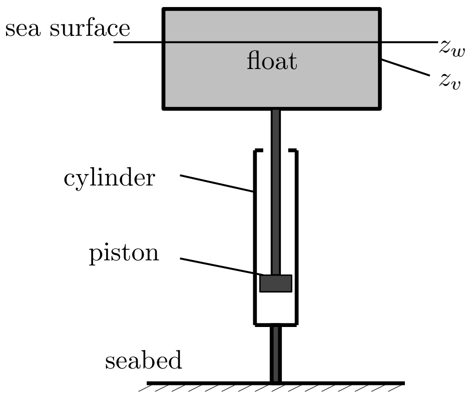


Figure 1. Schematic diagram of the point absorber

2018). Numerical simulations based on a typical WEC, called point absorber, are used to demonstrate the efficacy of the proposed APE and optimal control. In the simulations, the model used for the control design and parameter estimation are based on a typical second-order model and the WEC plant model is based on a high order model to demonstrate the robustness of the controller. Simulation results illustrates that the use of the estimated parameters in the optimal control can increase the energy output.

## 2. MODELLING OF WEC SYSTEM

A point absorber type of WEC to be studied in this paper has a float with a constant radius cylinder on the sea surface. Wave energy can be captured using different power take-off (PTO) mechanisms. Fig. 1 shows part of the potential hydraulic PTO design: a hydraulic cylinder is vertically installed below the float and is fixed to the bottom of the seabed. $z_w$ and $z_v$ are the water level and the heave displacement of the mid-point of the float respectively. The generator's torque is proportional to the force $f_u$ acting on the piston inside the cylinder. The extracted power is $P = -f_u v$, where the velocity on the piston is $v = \dot{z}_v$.

The dynamic equation for the float of a point absorber can be established using Newton's second law (Yu and Falnes, 1995)

$$m_s \ddot{z}_v = -f_s - f_r + f_e + f_u \tag{1}$$

where $m_s$ is the float mass, $z_v$ is the heave motion of the float.

The restoring force $f_s$ is given by

$$f_s = k_s z_v \tag{2}$$

where the hydrostatic stiffness coefficient is $k_s = \rho g s$, and $\rho$ is the water density, $g$ is the standard gravity, and $s$ as the cross-sectional area of the float.

The radiation force $f_r$ can be determined by

$$f_r = m_\infty \ddot{z}_v + \int_{-\infty}^{\infty} h_r(\tau)\dot{z}_v(t-\tau)d\tau \tag{3}$$

where $m_\infty$ is the added mass; $h_r$ is the kernel of the radiation force that can be computed via hydraulic software packages (e.g. WAMIT). The convolutional term in (3) $f_R := \int_{-\infty}^{\infty} h_r(\tau)\dot{z}_v(t-\tau)d\tau$ can be approximated by a causal finite dimensional state-space model $f_r = D_r(s)\dot{z}_v$, whose realization is given by

$$\dot{x}_r = A_r x_r + B_r \dot{z}_v \tag{4a}$$

$$f_r = C_r x_r \approx \int_{-\infty}^{t} h_r(\tau)\dot{z}_v(t-\tau)d\tau \tag{4b}$$

where $D_r(s) \sim (A_r, B_r\ C_r, 0)$ and $x_r \in \mathbb{R}^{n_r}$ are the state-space realization and the state respectively.

Following Yu and Falnes (1995), the wave excitation force $f_e$ can be determined by

$$f_e = \int_{-\infty}^{\infty} h_e(\tau) z_w(t-\tau)d\tau \tag{5}$$

where $h_e$ is the kernel of the radiation force and the state-space approximation is given by $f_e = D_e(s)z_w$, whose realization is given by

$$\dot{x}_e = A_e x_e + B_e z_w \tag{6a}$$

$$f_e = C_e x_e \approx \int_{-\infty}^{t} h_e(\tau) z_w(t-\tau)d\tau \tag{6b}$$

where $D_e(s) \sim (A_e, B_e\ C_e, 0)$ and $x_e \in \mathbb{R}^{n_e}$ are the state-space realization and the state respectively.

It is worth noticing that in this paper, to show the efficacy of this hierarchy adaptive optimal control, we intentionally create model mismatch by using static radiation and excitation coefficients $D_r$ and $D_e$ to approximate the dynamic coefficients (4) and (6) for demonstration purpose. This simplification also helps to reduce the computational burden of the controller to be developed. However, the proposed adaptive estimation and optimal control method proposed in this paper can be applied to higher order modeling without loss of generality.

By using the static radiation and excitation coefficients $D_r$ and $D_e$, the state-space model of a WEC for APE and optimal control design can be expressed by

$$\begin{cases} \dot{x} = A_c x + B_{uc} u + B_{wc} w \\ z = C_z x \end{cases} \tag{7}$$

where $w := z_w$, $z := \dot{z}_v$, $y := \dot{z}_v$, $x := [z_v, \dot{z}_v]$ and

$$A_c = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_s}{m} & -\dfrac{D_r(\delta_r)}{m} \end{bmatrix}\ B_{wc} = \begin{bmatrix} 0 \\ \dfrac{D_e(\delta_e)}{m} \end{bmatrix}\ B_{uc} = \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix}$$
$$C_z = [0\ \ 1] \tag{8}$$

with $m := m_a + m_\infty$. It is noted that structured parametric uncertainties $\delta_r$ and $\delta_e$ are present in radiation and excitation coefficients, respectively.

In the literature, by approximating the convolutional terms for calculating the radiation force (4) and excitation force (6) in state-space form, a higher order state-space model of the WEC system can be derived and used in the control design (Zhan and Li, 2018; Yu and Falnes, 1995). However, the growth of dimension in the model can lead to explosion of optimal control computational burden.

In this paper, to avoid the problem that comes from the model inaccuracy and sea wave variation, we present a hierarchical control algorithm for WECs where an APE is incorporated into the optimal control implementation. On the top layer, a recently proposed novel APE method (Na et al., 2015a) will be extended to estimate the time-varying coefficients of the radiation force and excitation force. With the real-time (online) estimation of the radiation and excitation force coefficients (e.g. $\delta_r$ and $\delta_e$), on the lower layer a recently reported linear noncausal optimal control (Zhan and Li, 2018) specifically developed for the WEC energy output maximization problem is adopted.

## 3. APE DESIGN OF FORCE GENERATION COEFFICIENTS

As shown in the above analysis, the uncertainties of WEC systems mainly come from the terms for calculating the forces $f_r$ and $f_e$, which appear in model (7) with unknown coefficients $D_r(\delta_r)$ and $D_e(\delta_e)$. Hence, the objective of this section is to introduce a new online APE method for estimating these two parameters based on the measured system state $x_2 = \dot{z}_v$, wave profile $w$ and control action $u$.

In order to achieve fast estimation convergence that is preferable for control design, we will further tailor a recently proposed APE method driven by the parameter estimation error information reported in (Na et al., 2015a). Hence, we rewrite the second equation of (7) as

$$\begin{aligned} \dot{x}_2 &= -\frac{k_s}{m}x_1 + \frac{1}{m}u - \frac{D_r}{m}x_2 + -\frac{D_e}{m}w \\ &= F(x,u) + \Phi(x,w)\Theta(t) \end{aligned} \tag{9}$$

where $F(x,u) = -\frac{k_s}{m}x_1 + \frac{1}{m}u$ is the known dynamics, $\Phi(x,w) = [-\frac{x_2}{m}, -\frac{w}{m}]$ is the regressor and $\Theta = [D_r, D_e]^T$ is the unknown force generation coefficients to be estimated.

*Assumption 1.* The derivative of unknown time-varying parameter vector $\Theta$ is bounded by $\left\|\dot{\Theta}(t)\right\| \leq \varpi$ for a positive constant $\varpi > 0$.

To design an adaptive law to online estimate $\Theta$, the following filtered variables of $x_2$, $w$, and $u$ are defined

$$\kappa\dot{x}_g + x_g = x_2, x_g(0) = 0, \tag{10a}$$

$$\kappa\dot{F}_g + F_g = F, F_g(0) = 0, \tag{10b}$$

$$\kappa\dot{\Phi}_g + \Phi_g = \Phi, \Phi_g(0) = 0, \tag{10c}$$

where $\kappa > 0$ is a small positive constant determining the bandwidth of the applied low-pass filter $1/(\kappa s + 1)$.

Now, we have the following results:

*Lemma 2.* : For system (9) with unknown parameter $\Theta$, the filter operations given in (10) are applied, then the variable $\beta = [(x_2 - x_g)/k - F_g - \Phi_g\Theta]$ is bounded and decreases exponentially for any finite $\kappa > 0$. Moreover, we know $\lim\limits_{k\to 0}[(x_2 - x_g)/k - F_g - \Phi_g\Theta] = 0$. Hence, the manifold $\beta = 0$ is an invariant manifold.

**Proof**: By applying a low-pass filter $1/(\kappa s + 1)$ on both sides of the second equation of (9), then we can have

$$\frac{s}{\kappa s + 1}[x] = \frac{1}{\kappa s + 1}[F] + \frac{1}{\kappa s + 1}[\Phi\Theta] \tag{11}$$

Consider the first equation of (10) (that is $\dot{x}_g = (x_2 - x_g)/\kappa$) and Swapping Lemma Sun and Sun (1995), one can represent (11) as

$$\dot{x}_g = \frac{x_2 - x_g}{\kappa} = F_g(x,u) + \Phi_g(x,w)\Theta(t) - \frac{\kappa}{\kappa s + 1}[\Phi_g\dot{\Theta}] \tag{12}$$

Since $\Phi(\cdot)$ is a smooth function of bounded variables $x, w$, then $\Phi_g$is bounded, i.e., $\|\Phi_g\| \leq \mu$ for a positive constant $\mu$. Moreover, it is assumed that $\left\|\dot{\Theta}(t)\right\| \leq \varpi$, so that for any finite $k > 0$, the term $\zeta = \frac{k}{ks+1}[\Phi_f\dot{\Theta}]$ is bounded (i.e., $\|\zeta\| \leq \gamma$ for a positive constant $\gamma$). Specifically, we know that $\lim\limits_{k\to 0}\zeta = 0$. Hence, $\zeta$ can be considered as a bounded disturbance perturbing the ideal manifold defined by $\beta$.

Hence, we can verify from (12) and (10) that

$$\dot{\beta} = -\frac{1}{\kappa}\beta - \dot{\zeta}. \tag{13}$$

where $\dot{\zeta}$ is also bounded as stated above.

Now, we choose a Lyapunov function as $V_\beta = \frac{1}{2}\beta^T\beta$, then its derivative can be calculated along (13) as

$$\dot{V}_\beta = \frac{1}{\kappa}\beta^T\beta - \beta^T\dot{\zeta} = \frac{1}{\kappa}V_\beta + \frac{\kappa}{2}\varrho^2, \tag{14}$$

where $\varrho = \dot{\zeta}^2$ denotes the bound of $\dot{\zeta}$. This further implies that $V_\beta(t) \leq e^{-\frac{t}{\kappa}}V\beta(0) + \frac{\kappa^2}{2}\varrho^2$ and thus $\|\beta(t)\| \leq \sqrt{\beta^2(0)e^{-\frac{t}{\kappa}} + \kappa^2\varrho^2}$. This implies exponential convergence of $\beta(t)$ to zero for $\kappa \to 0$, i.e., $\lim\limits_{k\to 0}[(x_2 - x_g)/k - F_g - \Phi_g\Theta] = 0$. Thus the manifold $\beta = 0$ is invariant. This completes the proof. □

It is shown in Lemma 1 that the manifold variable $\beta = [(x_2 - x_g)/k - F_g - \Phi_g\Theta]$ provides a reformulation of the information of the unknown parameter $\Theta$ based on the available variables $(x_2, x_g, F_g, \Phi_g)$. Hence, it can be used to design an adaptive law to online update the estimation $\hat{\Theta}$ of unknown parameters $\Theta$.

We define intermediate variables $M$ and $N$ as

$$\begin{cases} \dot{M} &= -\ell M + \Phi_g^T\Phi_g,\ M(0) = 0 \\ \dot{N} &= -\ell N + \Phi_g^T\left[\dfrac{(x_2 - x_g)}{\kappa} - F_g\right],\ N(0) = 0 \end{cases} \tag{15}$$

where $\ell > 0$ is another design parameter used to guarantee the boundedness of the induced varaible $M, N$.

Then, two vectors $W_1$ and $W_2$ can be calculated by

$$W_1 = M\hat{\Theta}(t) - N \tag{16}$$

$$W_2 = \Phi_f^T\Phi_f\hat{\Theta}(t) - \Phi_f^T[(x - x_f)/k - y_f] \tag{17}$$

where $\hat{\Theta}(t)$ is the estimate of unknown parameter $\Theta$.

Thus, the estimated parameter vector $\Theta$ can be updated by the following adaptive law

$$\dot{\hat{\Theta}} = -\Gamma(W_1 + \iota W_2) \tag{18}$$

where $\Gamma > 0$ is the learning gain, which can be set as a diagonal matrix, and $\iota > 0$ is a positive constant.

Before we prove the convergence of the proposed adaptive law (18), the following lemma is given

*Lemma 3.* (Na et al., 2015a): If the regressor vector $\Phi$ is persistently excited (PE), the matrix $M(t)$ is positive definite, and $\lambda_{\min}(M) > \sigma_1 > 0$) holds for a positive constant $\sigma_1$.

We refer to (Na et al., 2015a) for the proof of above lemma.

Now, the convergence of the APE (18) is given as

*Theorem 4.* : For system (9) with unknown parameter $\Theta$, the adaptive law (18) is used with the regressor matrix $\Phi$ being PE, then the estimation error $\tilde{\Theta}$ converges to a small set around zero.

**Proof**: We first solve the matrix equation (15), and then consider (11), (16) and (17)), then it follows

$$W_1 = -M\tilde{\Theta} + \psi \quad (19)$$

$$W_2 = -\Phi_f^T \Phi_f \tilde{\Theta} + \Phi_f^T \zeta \quad (20)$$

where $\tilde{\Theta} = \Theta - \hat{\Theta}$, $\psi = \int_0^t e^{-\ell(t-r)} \Phi_f^T(r)\zeta(r)dr$ is the residual error bounded by $\|\psi\| = \int_0^t e^{-\ell(t-r)} \Phi_f^T(r)\zeta(r)dr \leq \|\Phi_f\| \|\zeta\| / \ell = \mu\gamma/\ell$.

We choose a Lyapunov function as $V = \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta}/2$, and then calculate its derivative along (19) and (19) by using Young's inequality $a^T b \leq a^T a/2m + mb^T b/2$ for any constant $m > 0$, such that

$$\begin{aligned}\dot{V} &= -\tilde{\Theta}^T P \tilde{\Theta} + \tilde{\Theta}^T \psi - \iota \tilde{\Theta}^T \Phi_f^T \Phi_f \tilde{\Theta} + \iota \tilde{\Theta}^T \Phi_f^T \zeta + \tilde{\Theta}^T \Gamma^{-1} \dot{\Theta} \\ &\leq -(\sigma_1 - \frac{3}{2m}) \left\|\tilde{\Theta}\right\|^2 + \frac{m\mu^2\gamma^2}{2\ell^2} + \frac{m\mu^2\gamma^2\iota^2}{2} + \frac{m\varpi^2}{2\lambda_{\min}^2(\Gamma)} \\ &\leq -\alpha V + \rho \end{aligned} \quad (21)$$

where $\alpha = 2(\sigma_1 - 3/2m)/\lambda_{\max}(\Gamma^{-1}), \rho = m\mu^2\gamma^2(\iota^2 + 1/\ell^2)/2 + m\varpi^2/2\lambda_{\min}^2(\Gamma)$ are all positive constants for any design parameter $m \geq 3/2\sigma_1$. Then, from (21), one can obtain that $V(t) \leq e^{-\alpha t}V(0) + \rho/\alpha$ holds. $\tilde{\Theta}$ will converge to a small set around zero, whose size depends on the excitation level (e.g. $\sigma_1$), the residual error $\gamma$ and the adaptive gain $\Gamma$. □

*Remark 5.* According to (19) and (20), the introduced variables $W_1, W_2$ are the functions of the estimation error $\tilde{\Theta}$. Hence, they can be used to drive the adaptive law to achieve better estimation convergence performance. In particular, $W_2$ given in (17) contains the instant error information, in comparison to $W_1$ in (16) that introduces an 'average' effect by means of the filter $1/(s + \ell)$ in (15). Thus, the use of the instant error $W_2$ is important to estimate fast time-varying parameters compared to the algorithm suggested in (Na et al., 2015a) for constant parameters.

*Remark 6.* If the parameter $\Theta$ to be estimated is constant, we know $\dot{\Theta} = 0$, and thus the residual error $\zeta = \psi = 0$ and $\rho = 0$ in (21) are true. In this case, the estimation error $\tilde{\Theta}$ will exponential converge to zero.

## 4. LINEAR NONCAUSAL OPTIMAL CONTROL

Based on the on estimated excitation coefficients $D_e$ and radiation coefficient $D_r$, in this section, we will adopt a recently developed linear noncausal optimal control (LOC) strategy to maximise the energy conversion efficiency (Zhan and Li, 2018), which explicitly uses the wave prediction information provided by the wave prediction device. To design the noncausal LOC, we discretize the system (7) using the sampling time $T_s$ into

$$\begin{cases} x(k+1) = A_d x(k) + B_{ud} u(k) + B_{wd} w(k) \\ z(k) = C_z x(k) \end{cases} \quad (22)$$

Since the noncausal LOC can not explicitly cope with hard constraints, we construct the following cost function to penalize heave displacement $z(k)$ and control input $u(k)$ to avoid constraints violations

$$\sum_{k=0}^{N_p-1} (1/2)x^T(k)Qx(k) + u(k)z(k) + (1/2)ru^2(k) \quad (23)$$

Here

(1) $(1/2)x^T(k)Qx(k)$ is used to peneralize heave displacement $z(k)$ to prevent state constraint violation.
(2) $-u(k)z(k)$ represents the energy that can be captured by the WEC mechanism.
(3) $(1/2)ru^2(k)$ can be considered as a soft constraints for control input. The weight $r$ also influences the stability of the system.
(4) $N_p$ is the wave prediction horizon as well as the control horizon. The relationship between the control horizon $T$ and $N_p$ can be established by $T = N_p T_s$.

The adopted noncausal LOC problem of discrete time system (22) with objective function (23) for WECs is given by (Zhan and Li, 2018)

$$u_k = K_{x,k} x_k + K_{w,k} w_k + K_{s,k} s_{k+1} \quad (24a)$$

where

$$K_{x,k} = -(r + B_{ud}^T V_{k+1} B_{ud})^{-1}(C_z + B_{ud}^T V_{k+1} A_d) \quad (24b)$$

$$K_{w,k} = -(r + B_{ud}^T V_{k+1} B_{ud})^{-1} B_{ud}^T V_{k+1} B_{wd} \quad (24c)$$

$$K_{s,k} = -(r + B_{ud}^T V_{k+1} B_{ud})^{-1} B_{ud}^T \quad (24d)$$

and $V_k$ and $r_{k+1}$ can be calculated though the backward iterations

$$\begin{aligned} V_k = Q + A_d^T V_{k+1} A_d - (C_z + B_{ud}^T V_{k+1} A_d)^T (r \\ + B_{ud}^T V_{k+1} B_{ud})^{-1} (C_z + B_{ud}^T V_{k+1} A_d) \end{aligned} \quad (25a)$$

$$s_k = (A_d + B_{ud} K_{x,k})^T (V_{k+1} B_{wd} w_k + s_{k+1}) \quad (25b)$$

with the boundary conditions $V_{N_p} = 0$, $s_{N_p} = 0$.

The detailed analysis of this new LOC is omitted due to the page limit. For those interested readers, we refer to (Zhan and Li, 2018) for more details.

The proposed optimal control consists of the following three parts:

- the linear state feedback part $K_{x,k} x_k$ depending on the current system state.
- the feedforward part $K_{w,k} w_k$ depending on the current wave measurement.
- the anti-causal part $K_{s,k} s_{k+1}$, where $s_{k+1}$ depends only on the future wave information provided by some wave prediction technique.

The calculation of the optimal controller (24) can be implemented by computing though the backward iteration of (25a) for the gains $K_{x,k}$, $K_{w,k}$ and $K_{s,k}$ then followed by another backward iteration (25a) for $s_k$ incorporating the wave prediction information. The online backward recursion result $s_1$ can be viewed as the accumulation of the impact from the incoming wave prediction on the optimal control action (Zhan and Li, 2018).

## 5. SIMULATIONS

In this section, we use the point absorber described in Section 2 as an example to show the efficacy of the proposed adaptive optimal control. The WEC model and controller model are chosen in the following cases.

*Case 1 - No model mismatch :* Both of the WEC model and controller model are with the state-space matrices as

(8) and the same coefficients as $k_s = 3.8658 \times 10^3$ N/m, $m = 325.5$ kg, $D_r = 400$ Ns/m, and $D_e = 2660$ Ns/m.

*Case 2 - With parametric uncertainties:* Both of the WEC model and controller model with the state-space matrices as (8). The controller model damping terms are changed to $D_r = 200$ Ns/m, and $D_e = 2000$ Ns/m, which represents parametric uncertainties.

To validate the convergence of the proposed APE, and show the efficacy of LOC with APE, the following three scenarios are compared:

1) **No model mismatch:** Modeling Case 1 and the APE is not necessary.
2) **Without APE:** Modeling Case 2 and no APE is used.
3) **With APE:** Modeling Case 2 and the APE is used.

The weights of the LOC cost function are tuned as $Q = \mathrm{diag}(4 \times 10^3, 0)$, $r = 2 \times 10^{-4}$ to tradeoff the constraints on the output $x_1$ and control action $u$, and the generated energy. The other parameters used for the APE are set as $\kappa = 0.08$, $\ell = 1$, $\Gamma = \mathrm{diag}(5500, 5000)$, $\iota = 50$.

A segment of realistic sea wave heave profile for a period of 50 s, gathered off the coast of Cornwall, UK, is used in the simulations. The magnitude of this wave profile as shown in Fig. 2 is small and its maximum heave magnitude is less than 2 m. Figs. 3-4 provide the profiles of the proposed AEP. Fig. 3 shows that the assumed correct damping terms $D_r = 400$ Ns/m, and $D_e = 2660$ Ns/m associated with the radiation force and wave excitation force can be precisely estimated after a short transient time within 15 seconds. With these estimated coefficients, the radiation force $f_r$ and excitation force $f_e$ can be estimated as shown in Fig. 4. Hence, these correctly estimated coefficients can help to improve the overall control response. One can find from Figs. 5-6 that with the selected control parameters (e.g. $r, Q$), there are no significant differences between the results in scenario 1) and scenario 3) in terms of state trajectories, the required control actions (Fig. 5) and the generated power (Fig. 6). In particular, the extracted energy of Case 3) ($2.9437 \times 10^4$ J with APE) is almost the same as that of Case 1) ($2.9438 \times 10^4$ J no model mismatch). However, for the scenario 2) when the APE is not used, the extracted energy is reduced to $2.8051 \times 10^4$ J as shown in Fig. 6. These results demonstrate the robustness of the proposed control framework combining the APE and LOC under the model mismatch.

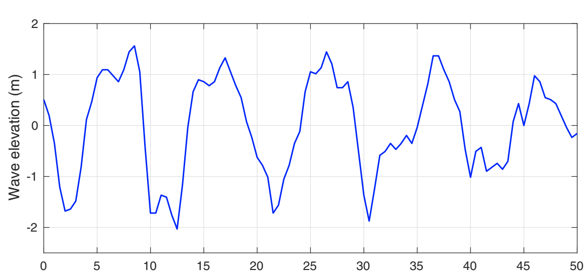


Figure 2. 50 s of wave profile used for Simulation.

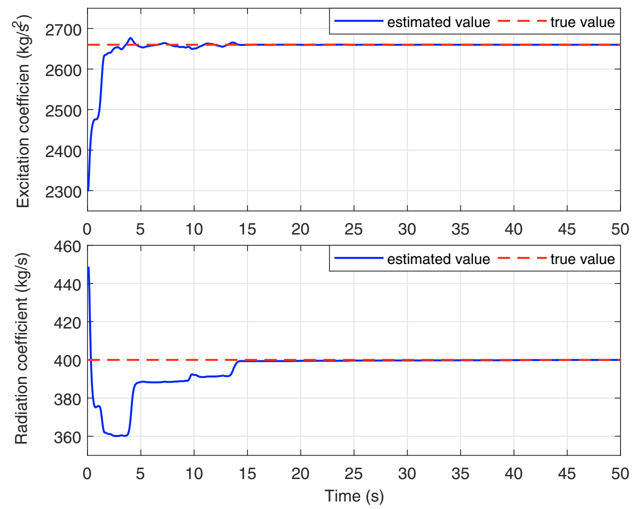


Figure 3. Estimation of the radiation force coefficient $D_r$ and excitation force coefficient$D_e$.

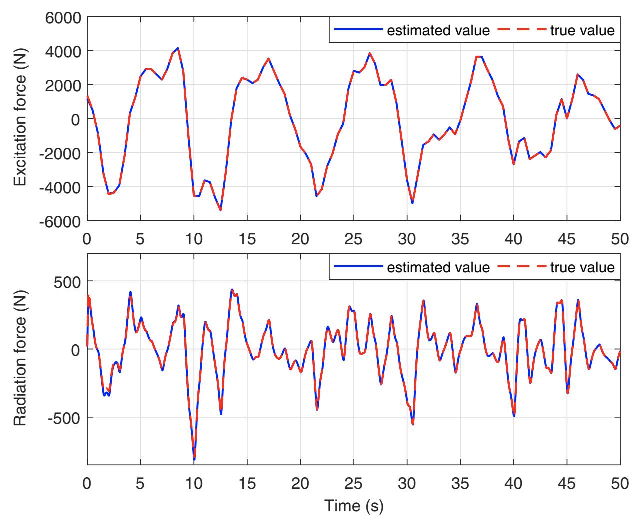


Figure 4. Estimation of radiation and excitation forces.

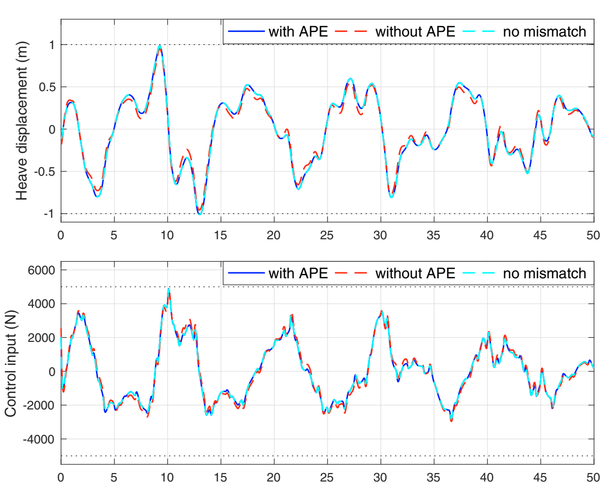


Figure 5. Trajectories of states and control inputs for 3 cases: 1) No model mismatch, 2) with model mismatch and no APE, 3) with model mismatch and APE.

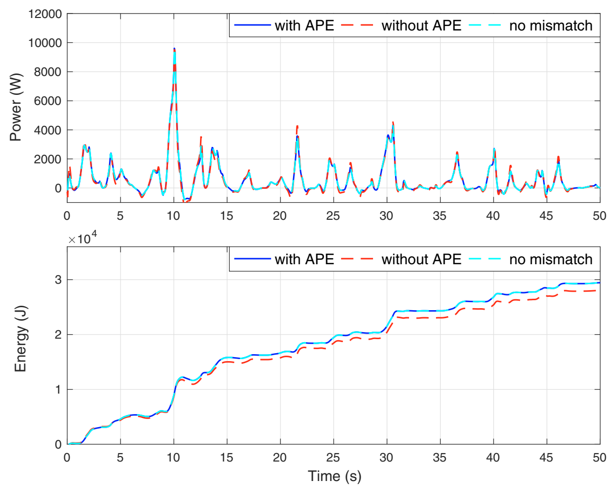


Figure 6. Energy outputs for 3 cases: 1) No model mismatch, 2) with model mismatch and no APE, 3) with model mismatch and APE.

## 6. CONCLUSION

A new hierarchical adaptive optimal control framework is proposed for WECs by combining an adaptive parameter estimation and a linear optimal control. A new adaptive law with fast and guaranteed convergence is developed based on a benchmark second-order WEC to estimate the unknown damping terms associated with excitation force and radiation force. Then the estimated damping coefficients are incorporated into a linear noncausal optimal control specifically developed for WEC control. Thus the proposed framework combines the strength of optimal control for generating maximum energy output and APE in coping with model parameter variations. Simulations clearly demonstrate the efficacy of the proposed adaptive optimal control method.

## ACKNOWLEDGEMENTS

This work was supported by a Newton Mobility Grant, jointly funded by the Royal Society and NSFC (No. IE150833/6151101245), a NSFC grant (No. 61573174), and a research contract from Wave Energy Scotland's Control Systems programme.

## REFERENCES

Babarit, A. and Clément, A.H. (2006). Optimal latching control of a wave energy device in regular and irregular waves. *Applied Ocean Research*, 28(2), 77–91.

Babarit, A., Guglielmi, M., and Clément, A.H. (2009). Declutching control of a wave energy converter. *Ocean Engineering*, 36(12–13), 1015–1024.

Budal, K., Falnes, J., Hals, T., Iversen, L.C., and Onshus, T. (1981). Model experiment with a phase controlled point absorber.

Bull, D. and Ochs, M.E. (2013). Technological cost-reduction pathways for attenuator wave energy converters in the marine hydrokinetic environment. Technical report, Sandia National Laboratories. SAND2013-7207, Unlimited Release, Sandia Report.

Clément, A. and et al (2002). Wave energy in europe: current status and perspectives. *Renewable and Sustainable Energy Reviews*, 6(5), 405 – 431.

Ding, F., Xu, L., and Zhu, Q. (2016). Performance analysis of the generalised projection identification for time-varying systems. *Iet Control Theory & Applications*, 10(18), 2506–2514.

Falnes, J. (2002). *Ocean Waves and Oscillating Systems*. Cambridge University Press.

Glendenning, I. (1977). Ocean wave power. *Applied Energy*, 3(3), 197 – 222.

Korde, U.A. (2002). Latching control of deep water wave energy devices using an active reference. 29(11), 1343–1355.

Li, G., Weiss, G., Mueller, M., Townley, S., and Belmont, M.R. (2012). Wave energy converter control by wave prediction and dynamic programming. *Renewable Energy*, 48(48), 392–403.

Na, J., Mahyuddin, M.N., Herrmann, G., Ren, X., and Barber, P. (2015a). Robust adaptive finite-time parameter estimation and control for robotic systems. *International Journal of Robust and Nonlinear Control*, 25(16), 3045 – 3071.

Na, J., Yang, J., Ren, X., and Guo, Y. (2015b). Robust adaptive estimation of nonlinear system with time varying parameters. *International Journal of Adaptive Control & Signal Processing*, 29(8), 1055–1072.

Nielsen, S.R., Zhou, Q., Kramer, M.M., Basu, B., and Zhang, Z. (2013). Optimal control of nonlinear wave energy point converters. *Ocean Engineering*, 72, 176 – 187.

Samandeep Dahliwal, M.G. (2014). Set-based adaptive estimation for a class of nonlinear systems with time varying parameters. *Journal of Process Control*, 24(2), 479–486.

Scruggs, J.T., Lattanzio, S.M., Taflanidis, A.A., and Cassidy, I.L. (2013). Optimal causal control of a wave energy converter in a random sea. *Applied Ocean Research*, 42(42), 1–15.

Sun, J. and Sun, J. (1995). *Robust adaptive control*. Prentice-Hall, Inc.

Yu, Z. and Falnes, J. (1995). State-space modelling of a vertical cylinder in heave. *Applied Ocean Research*, 17, 265–275.

Zhan, S., Huijberts, H., Na, J., and Li, G. (2016). Optimal controller design and constraints analysis of a sea wave energy converter. In *UKACC*. Belfast, UK.

Zhan, S. and Li, G. (2018). Linear optimal noncausal control of wave energy converters. *IEEE Transactions on Control Systems Technology*, (Available Online).